Criteria of morphometric analysis of a daily load profile, International Transactions on Electrical Energy Systems

Komenda, T., Komenda, N., and Vagapov, Y.

Criteria of Morphometric Analysis of a Daily Load Profile

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Summary

Analysis of electrical loads is crucial for a proper operation and control of electrical energy sources and planning and design of electrical power systems in terms of optimum capacity of the electricity generation. A typical daily load profile significantly varies over the 24-hour day and requires levelling actions which can be advised from the detailed analysis of the profile. This paper discusses the principles and implementation of morphometric analysis for a daily load profile evaluation using three criteria: roundness, compactness, and elongation. In order to conduct the morphometric analysis, the daily load profile represented as a time series has to be converted into a polygon of a particular form in a radar chart. The criteria for the profile analysis are based on geometrical interpretation of the shape of the polygon. The criterion roundness assesses the maximum and minimum loads of the profile and are related to the ratio of the inner circle of the polygon to the outer circle. The criterion compactness is based on the polygon perimeter and its inner area. The criterion elongation is defined as a relationship between the length of perpendicular to main axis of the polygon and the length of the main axis. The examples of the load profiles represented as a regular polygon and the case study have been used to demonstrate implementation of the analysis using the roundness, compactness, and elongation. It has been shown that the analysis using the morphometric criteria can be effectively applied for the detailed assessment of the load profiles.

Keywords

Daily load profile, Morphometric analysis, Roundness, Compactness, Elongation

1 Introduction

The balance between electrical energy generation and load consumption is a fundamental principle of operation of electrical power system. Effective control of power sources integrated into the power grid significantly depends on the load variation and its prediction. Analysis and understanding of the consumers' load profiling are extremely important for a proper operation of controlled electrical energy sources and implementation of smart grid technologies in particular.

The prediction of load profile is crucial for planning and design of electrical power systems in terms of optimum capacity of the electricity generation [1-4].

Daily load profile attracts particular attention in the context of power system analysis as it represents a periodic pattern of electrical load demand. A typical daily load profile demonstrates a significant variation of power consumption over 24 hours of day time. This variation reflects a non-uniform character of power demand related to on-peak and off-peak loads. Such a load profile non-uniformity brings a negative impact on electrical energy generation and distribution [5-7]. A typical load-levelling approach applied for “peak shaving” and “valley filling” of daily load profile usually aggregates a number of solutions aimed to control the consumption at the load side. These are the implementation of energy storage, load time shift scheduling, reduction of energy consumption etc. However, the load levelling for large industrial consumers is quite complicated due to reduced efficiency of these solutions for high power loads. For example, the load time shift could significantly affect the production process of an industrial manufacturer whereas a high-power storage system could be unaffordable for this manufacturer due to its high cost. Therefore, the implementation of the load levelling actions leading to an industrial load control must be thoroughly assessed and justified. This requires a detailed analysis of the uniformity of the daily load profile [8-10].

There are many methods and tools for load profile assessment and analysis. Majority of these methods are based on analysis of the load profile represented as time series (load power against time) [5-8]. However, this paper focuses on the morphometric method of the load profile analysis proposed in [11]. This method is based on the approach that the daily load profile, originally represented as a *xy*-plot, (time series) is converted into a radar chart and shown as a polygon of a particular form. The paper discusses the criteria applied for morphometric analysis of the daily load profile in order to provide a thorough assessment of power consumption. The results of analysis can be used for the further load management and forecast.

2 Morphometric Analysis

A morphometric approach is a very effective method widely used for shape and image analysis in many areas of science and engineering. For example, the method is successfully applied in signal processing for image and video filtering, segmentation, and feature extraction [12,13]. It is also used in the map related studies for analysis of morphometric parameters such as size and shape of topographical elements [14,15]. Biology and medical science are also areas where morphometric methods provide effective assessment of statistical data in the context of size and shape of variables [16,17].

Implementation of the morphometric method in the area of electrical power engineering for the daily load profile analysis requires conversion of the profile into a radar chart where the time is represented as an angle (in this paper, 24 readings taken each hour are corresponding to 360 degrees in the radar chart) and the load power is shown as the length of the vector from the zero reference point of the chart. Due to this conversion the load profile as is introduced as a polygon of a particular form. The polygon as a geometrical object is applicable for the analysis using morphometric criteria [11]. Fig. 1 shows an example of the conversion of the load profile from time series representation into a polygon in the radar chart.

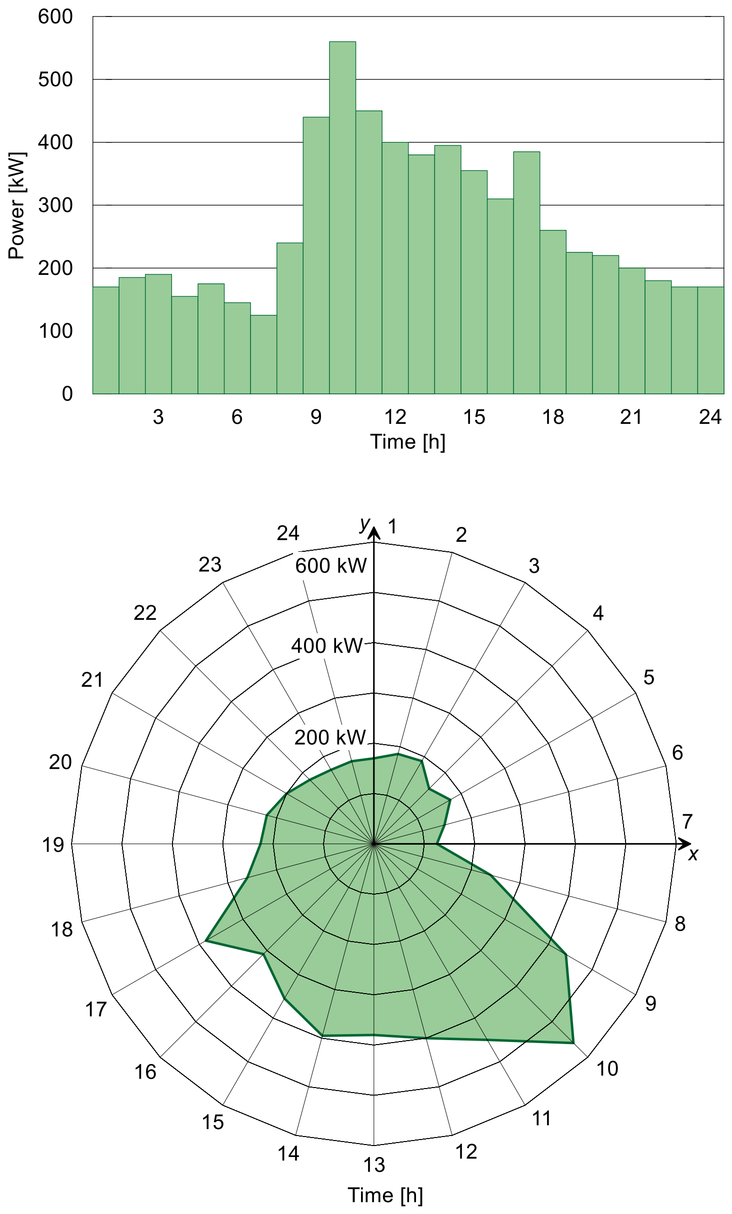


Fig. 1. A radar chart representing a daily load profile as a polygon.

In fact, the load profile can be converted into a radar chart using any number of readings of the load power; for example, if the time series of the load profile is composed of 48 readings taken every 30 minutes, the radar chart will be segmented by 360°/48 = 7.5 degrees.

The drawback of the morphometric method is that it can not be applied for the analysis of the load having a negative value – the radar chart of the load profile is not designed to show negative parameters. Therefore, the morphometric method is not suitable for analysis of active electrical loads having, for example, sources of renewable energy or energy storages where such loads can generate the electricity into the grid and demonstrate a negative power consumption. This is why this method is applicable for analysis of conventional load only where a customer consumes electrical power from the grid but not generates it.

Although the polygon is built in the radar chart reference frame the morphometric method needs geometrical description of the polygon in accordance to *x*,*y* coordinates represented in the chart as conventional *x* and *y* axis. Therefore, the polygon shown in Fig. 1 is defined using the following geometrical parameters: perimeter; area and coordinates of weight centre of the polygon.

Perimeter of the polygon P is sum of the lengths of all segments formed the figure [18]:

$$P = \sum_{i=1}^{m} l_i = \sqrt{(x_m - x_1)^2 + (y_m - y_1)^2} + \sum_{i=1}^{m-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{1}$$

where l_i is the length of *i*-th segment, $m = 24$ is number of readings of the load profile.

The area of the polygon S is defined as absolute value of A as following:

$$S = |A| \tag{2}$$

where

$$A = \frac{1}{2}\left[(x_m y_1 - x_1 y_m) + \sum_{i=1}^{m-1} (x_i y_{i+1} - x_{i+1} y_i) \right]$$

The coordinates of the polygon weight centre x_c and y_c are defined as:

$$\begin{cases} x_c = \dfrac{1}{6A}\left[(x_m + x_1)(x_m y_1 - x_1 y_m) + \sum_{i=1}^{m-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \right] \\ y_c = \dfrac{1}{6A}\left[(y_m + y_1)(x_m y_1 - x_1 y_m) + \sum_{i=1}^{m-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \right] \end{cases} \tag{3}$$

The perimeter, area and coordinates of weight centre of the polygon are the key parameters in the morphometric method and used for development of the criteria for the daily load profile assessment procedure. The morphometric criteria introduced in work [11] are roundness (M_1), compactness (M_2), and elongation (M_3). However, this work has a lack of theoretical background of the criteria development and does not provide the detailed discussion on how to implement the criteria for the thorough analysis. In the authors' best knowledge there are no other published papers investigating the morphometric method criteria for the area of electrical power engineering and the load profile analysis in particular. The aim of this paper is to give the theoretical explanations and understanding of the criteria integration in the analytical procedure of the daily load profile assessment. This paper produces a detailed explanation and discussion on implementation of three morphometric method criteria and underpins it with examples and a case study.

3 Roundness Criterion

According to the definition [11], the criterion roundness (M_1) is applied to estimate the relationship between minimum and maximum values of the load and used to characterise the extremum of the daily electrical energy consumption process. This criterion is linked to the shape of the polygon obtained from representation of the load profile as the radar chart. For example, if a daily profile is uniform, a polygon in the radar chart has a circle form.

Roundness (M_1) is determined as the ratio of the radius of the inner circle to the radius of outer circle outlining the polygon as shown in Fig. 2. In fact, the centres of both circles are corresponding the centre of the polygon weight.

$$M_1 = \frac{R_{\min}}{R_{\max}} \tag{4}$$

The coordinate offset of the polygon weight centre x_c, y_c from the radar chart coordinates centre x_0, y_0 characterises the daily load profile non-uniformity. If the profile is uniform, then the polygon has a circle form with coordinates $x_c = x_0$; $y_c = y_0$. Otherwise, there is an offset d which value is related to the load profile non-uniformity (Fig. 2). The offset d is defined as [18]

$$d = \sqrt{\left(x_0 - x_c\right)^2 + \left(y_0 - y_c\right)^2} \tag{5}$$

The improvement of the profile uniformity reduces the profile polygon to a circle where the weight centre is approaching the coordinate centre and the offset is reduced to 0: $d \to 0$. Therefore, d estimates the polygon non-uniformity and reflects any peaks and hollows of the daily load profile. The criterion roundness is approaching 0 if there is a large difference between the radii (if $R_{\min} << R_{\max}$, then $M_1 \to 0$). On the other hand, if the radii are closer each other, then the criterion roundness is approaching 1 (if $R_{\min} \approx R_{\max}$, then $M_1 \to 1$).

It can be seen in Fig. 2 that M_1 gives a detailed estimation of the uniformity of the load profile, which is similar to a criterion $K_n = P_{\min}/P_{\max}$ – non-uniformity index [19]. However, the criterion M_1 is integral characteristic whereas K_n takes into account just two values – maximum and minimum loads of the daily profile.

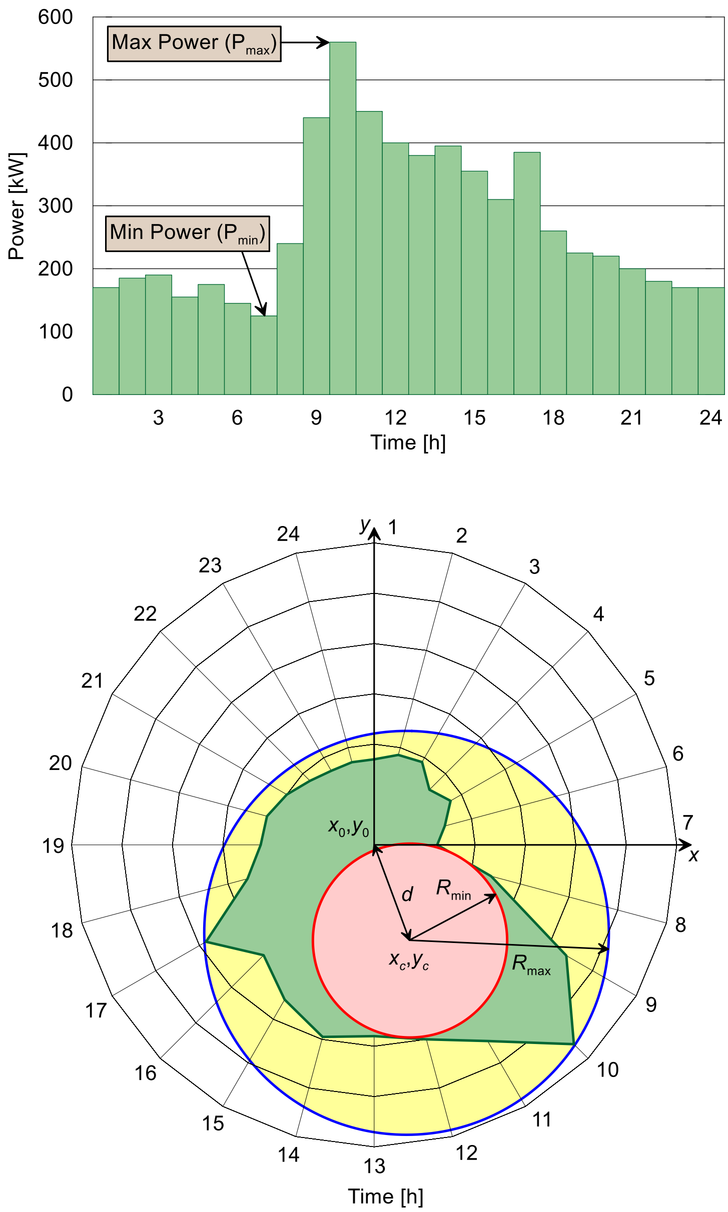


Fig. 2. A radar chart of a daily load profile as a polygon. Two circles – R_{min} (the inner circle radius, red colour) and R_{max} (the outer circle radius, yellow colour) – illustrate the application of the criterion roundness.

The complicated shape of the polygon makes it difficult to calculate R_{min} and R_{max}. In order to simplify the analysis using M_1, the profile is considered as a regular polygon in which all angles and all sides are equal. Fig. 3 shows an example of regular polygon called heptagon.

The inner radius of the regular heptagon shown in Fig. 3 can be found as [18] $R_{min} = R_{max} \cos\left(\frac{\pi}{n}\right)$, thus the ratio $\frac{R_{min}}{R_{max}} = \cos\left(\frac{\pi}{n}\right)$. Therefore, the criterion roundness M_1 for regular polygon is defined as

$$M_1 = \cos\left(\frac{\pi}{n}\right) \tag{6}$$

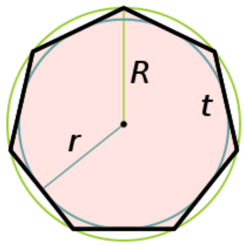
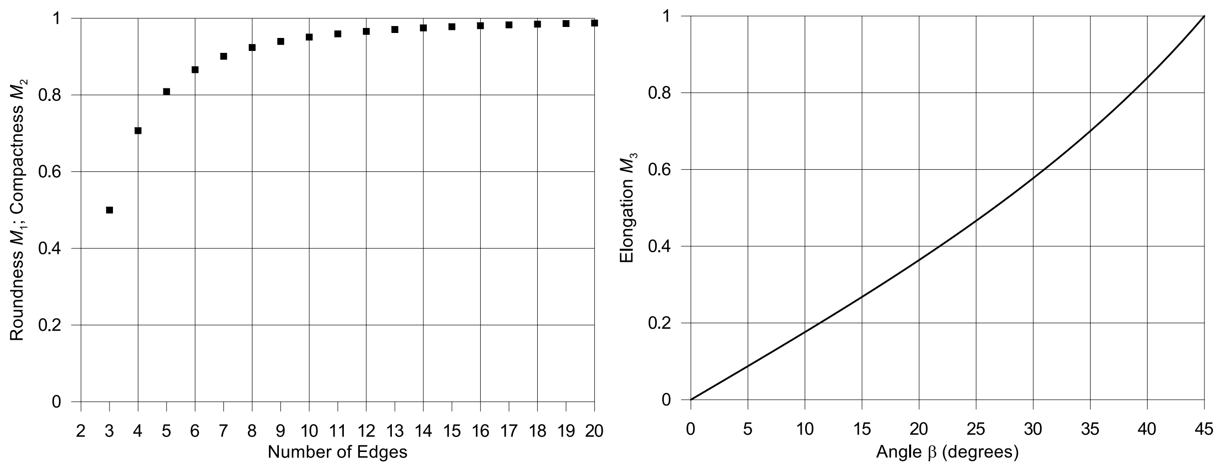


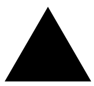
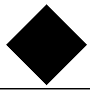
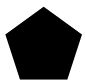
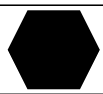
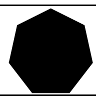
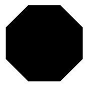
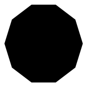
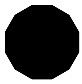
Fig. 3. Regular heptagon, where $R = R_{max}$ is the outer circle radius; $r = R_{min}$ is the inner circle radius, t is heptagon edge length, $n = 7$ is the number of the heptagon edges.



(a) (b)

Fig. 4. (a) Plot of coundness M_1 and compactness M_2 against the number of polygon edges, (b) Plot of elongation M_3 against angle β.

Table 1. Analysis of M_1 and M_2 for the simple regular polygons.

Number of Edges	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 10$	$n = 12$
Polygon form								
$\frac{\pi}{n}$	1.05	0.79	0.63	0.52	0.45	0.39	0.31	0.26
$M_1 = \cos\left(\frac{\pi}{n}\right)$	0.5	0.7	0.81	0.87	0.9	0.92	0.95	0.97
$M_2 = \frac{\pi}{n}\tan^{-1}\left(\frac{\pi}{n}\right)$								
$M_1' = -\sin\left(\frac{\pi}{n}\right)$	−0.86	−0.7	−0.59	−0.5	−0.43	−0.38	−0.3	−0.26
$M_2' = -\sin\left(\frac{\pi}{n}\right)$								

Since $n > 2$ (the shape of the profile is a regular polygon), the argument of the cosine is always positive and has the value within the boundaries of (0; π/2). The argument depends upon the polygon form and reflects the uniformity using the criterion M_1. Fig. 4 shows the plot of $M_1 = \cos(\pi/n)$ over the region $n \in [2; 20]$. The simple cases of the roundness analysis (M_1) for regular polygons are given in Table 1.

Fig. 4a and Table 1 demonstrates that the value of M_1 approaches 1 if $n \to \infty$ ($\pi/n \to 0$). Under this condition, the form of the polygon is reducing to the form of a circle. Table 1 also shows a number of values for the derivative of $M_1 = \cos(\pi/n)$ which actually reflects the speed of the function change. It can be seen that the bigger values of the derivative are corresponding to smaller number of polygon edges n.

If the value of roundness M_1 is known, the number of the regular polygon edges (and character of the non-uniformity) can be determined using the following equation.

$$n = \frac{\pi}{\cos^{-1}(M_1)}. \tag{7}$$

The following two examples demonstrate the use of roundness M_1 for the analysis of the load profile uniformity.

Example 1. There are two industrial electricity consumers with different daily load profiles. One consumer has the load profile having a regular polygon with 8 edges ($n = 8$). The second consumer has the load profile of the form of a regular polygon with 7 edges ($n = 7$). The cumulative load uniformity should be found at the node that feeds both consumers. It is assumed that the additional load peaks will not be overlapped and the total profile is a regular polygon. From Table 1, roundness for the first consumer is $M_1 = 0.92$ whereas the roundness for the second consumer is $M_1 = 0.9$. The cumulative load profile in the node will be characterised by $n = 8 + 7 = 15$. It means that the profile has a form of regular polygon with 15 edges. Therefore, the uniformity of cumulative load in the node is $M_1 = \cos(\pi/n) = \cos(\pi/15) = 0.98$. It means that the daily load profile at the node demonstrates greater uniformity and this is reflected by the increase in the roundness.

Example 2. There are two industrial electricity consumers supplied from the same node. One consumer has a daily load profile in the form of regular polygon with 3 edges. The load profile of the node is a regular polygon with 10 edges. The task is to find the uniformity of the second consumer assuming that the addition load peaks will not be overlapped and the resulting profile is a regular polygon. From Table 1 the roundness for the first consumer is $M_1 = 0.5$ whereas the roundness for the node is $M_1 = 0.95$. The load profile for the second consumer is characterised by $n = 10 – 3 = 7$. This means that the profile has a form of regular polygon with 7 edges. Therefore, the uniformity of the load of the second consumer is defined as $M_1 = \cos(\pi/n) = \cos(\pi/7) = 0.9$. This demonstrates that the second consumer load profile has less uniformity than the node due to decrease in the roundness.

The given examples are obviously simple and intended just to show the principles roundness usage for the analysis.

4 Compactness Criterion

The next criterion compactness M_2 is defined as the ratio between the area of the load profile S and its perimeter squared P^2 as it shown in Fig. 5 [11].

$$M_2 = \frac{4\pi S}{P^2} \tag{8}$$

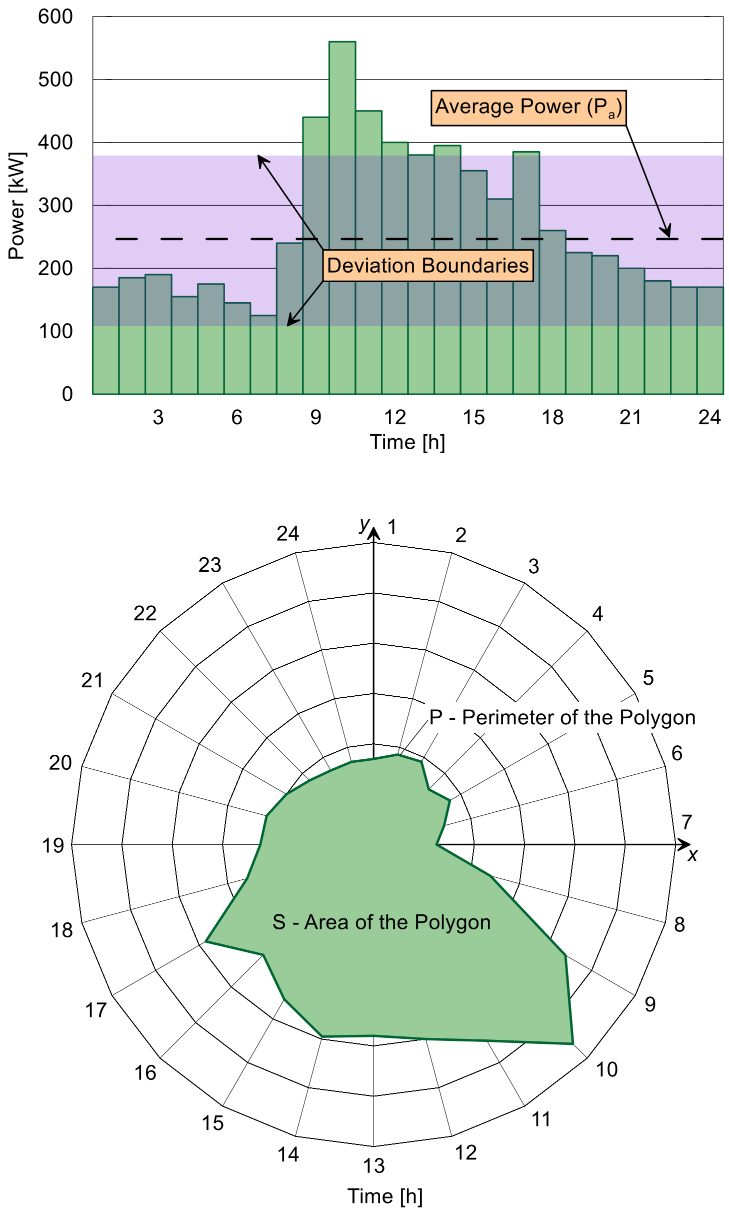


Fig. 5. Perimeter and area of the polygon of the radar chart for the daily load profile. Both the perimeter and area are used to calculate compactness M_2

In case of non-uniformity of the profile the value of the polygon perimeter P is significantly increased whereas the value of the polygon area S could be constant. Under this condition the criterion compactness $M_2 \rightarrow 0$. In the opposite case, when the polygon of the load profile is uniform and formed a circle, the area is defined as $S = \pi R^2$ and the perimeter is $P = 2\pi R$. The substitution of these values into (8) produces the following expression showing that the optimum value of compactness $M_2 = 1$.

$$M_2 = \frac{4\pi^2 R^2}{(2\pi R)^2} = \frac{4\pi^2 R^2}{4\pi^2 R^2} = 1$$

The area S and the polygon perimeter P can be found using (1) and (2). The calculation of the area S and the perimeter P of the complicate shape polygon requires more computational resources. To simplify analysis, and demonstrate advantages of using compactness M_2, a daily load profile is represented as a regular polygon having equal angles and sides (Fig. 3). This assumption makes easier analytical investigation of compactness M_2. The compactness criteria M_2 is linked to the deviation of the varying load profile σ relatively the average value P_{AVER} which is a conventional index very often applied to the analysis of a power load.

The area of a regular polygon S is defined as $S = \frac{P}{2} R_{\min}$ [18], where P is the perimeter of the polygon and $R_{\min}$ is the inner radius of the polygon (Fig. 3). The substitution of the area into (8) gives the following expression.

$$M_2 = \frac{4\pi S}{P^2} = \frac{4\pi \frac{P}{2} R_{\min}}{P^2} = \frac{2\pi R_{\min}}{P} \tag{9}$$

If the shape of the polygon is a circle then $R = R_{\max} = R_{\min}$. Therefore $M_2 = 1$.

It is known [18] that the inner radius for regular polygons is $R_{\min} = \frac{a}{2\tan\left(\frac{\pi}{n}\right)}$ and the perimeter is $P = na$, where n is the number of the polygons edges, and a is the length of an edge. The compactness M_2 can be modified by substituting the inner radius of a regular polygon into (9).

$$M_2 = \frac{2\pi R_{\min}}{P} = \frac{2\pi \frac{a}{2\tan\left(\frac{\pi}{n}\right)}}{na} = 2\pi \frac{a}{2\tan\left(\frac{\pi}{n}\right)} \times \frac{1}{na} = \frac{\pi}{n\tan\left(\frac{\pi}{n}\right)} = \frac{\pi}{n}\tan^{-1}\left(\frac{\pi}{n}\right) \tag{10}$$

Therefore, the equation for compactness M_2 has the following form.

$$M_2 = \frac{\pi}{n}\tan^{-1}\left(\frac{\pi}{n}\right) \tag{11}$$

Since $n > 2$ (the shape of the profile is a regular polygon), the argument of the cotangent is always positive and has the value within the boundaries of $(0; \pi/2)$. The argument reflects the form of the polygon using the uniformity criterion M_2. Fig. 4a shows the plot of $M_2 = \frac{\pi}{n} \cdot \tan^{-1}\left(\frac{\pi}{n}\right)$ over the region $n \in [2; 20]$; $\pi/n \in [\pi/20; \pi/2]$. The simple cases of the compactness analysis (M_2) for regular polygons are given in Table 1. Fig. 4a and Table 1 show that the value of M_2 is approaching to 1 if $n \to \infty$ ($\pi/n \to 0$). Under the condition $n \to \infty$, the shape of the regular polygon is reduced to the form of a circle. The derivative values of $M_2 = \frac{\pi}{n} \cdot \tan^{-1}\left(\frac{\pi}{n}\right)$ are given in Table 1 to reflect the speed of the function change. It can be seen that the greater values of the derivative are corresponding to smaller number of polygon edges n.

The number of the regular polygon edges and, therefore, the profile uniformity can be found from the criterion compactness M_2.

$$n = \frac{\pi}{\tan\left(M_2 \frac{n}{\pi}\right)} \tag{12}$$

5 Elongation Criterion

Criterion elongation M_3 is defined as the relationship between the length of perpendicular to main axis of the polygon representing a daily load profile L_2 and the length of the main axis L_1 as shown in Fig. 6. The main axis is determined as longest axis that crosses the weight centre (3) of the polygon (x_c,y_c).

$$M_3 = \frac{L_2}{L_1} \tag{13}$$

The criterion elongation M_3 is approaching 0 if the main axis length L_1 is much longer than the axis length L_2 (if $L_2 << L_1$, then $M_3 \to 0$). On the other hand, if the length of the axes are closer each other, then the criterion roundness is approaching 1 (if $L_2 \approx L_1$, then $M_3 \to 1$).

The peak loads matched the peaks of the power system can be determined using analysis of the angle α (Fig. 6). It can be seen that one hour of load profile is equal to 360/24 = 15° of the main axis rotation. In general, the time shift of the peak load from 7 am in the morning can be found as following.

$$t = \frac{\alpha}{15}, \tag{14}$$

where α is elongation axis angle.

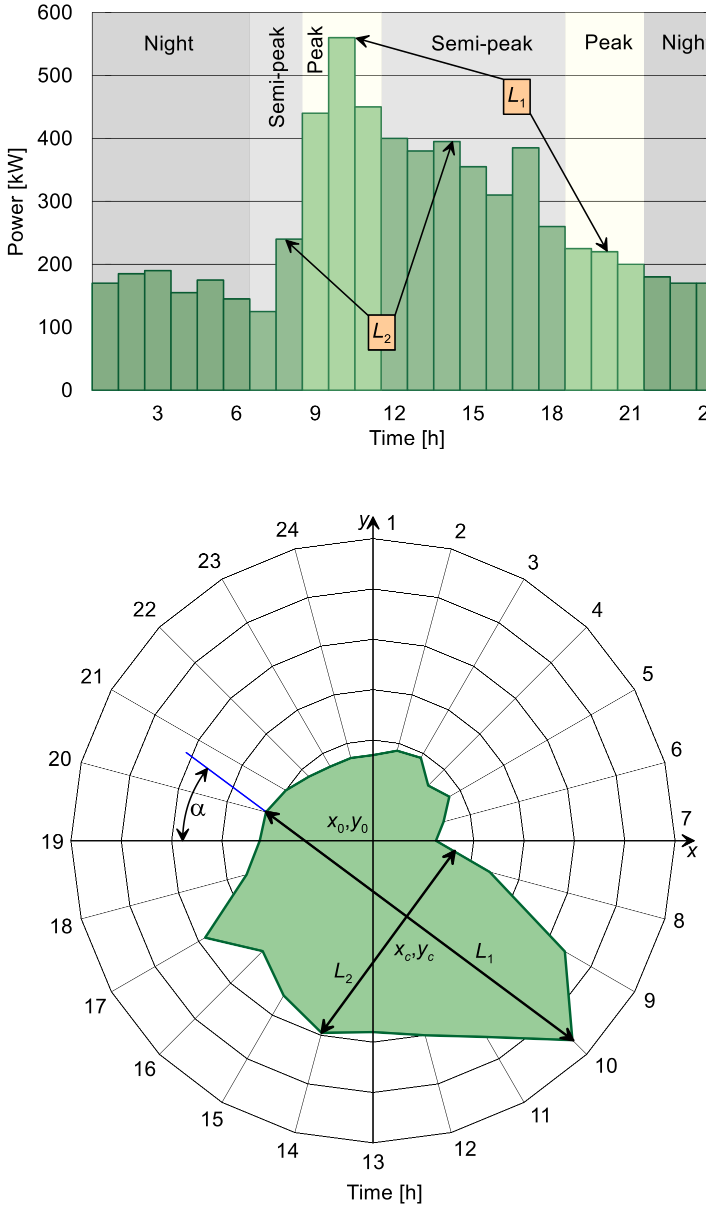


Fig. 6. Illustration of the lengths L_1 and L_2 to calculate the elongation criteria M_3.

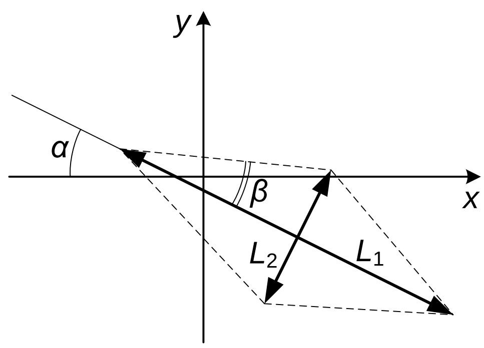


Fig. 7. Simplified plot for analysis of elongation parameters in the rhombus.

Table 2. Analysis of M_3 for the simple regular polygons.

β	5.63°	6.43°	7.5°	9°	11.25°	15°	22.5°	45°
$M_3 = \tan\beta$	0.1	0.11	0.13	0.16	0.20	0.27	0.41	1.00
$M_3' = \frac{1}{\cos^2\beta}$	1.01	1.01	1.02	1.03	1.04	1.07	1.17	2.00

If the time shift $t \in [9,11] \vee t \in [19,21]$ then a peak load is in the peak zone of the power system load. If $t \in [22,6]$ then a peak load is in the night zone of the power system load. In other cases, a peak load is allocated in the semi-peak hours of the power system load (Fig. 6).

Fig. 7 shows the lengths of two axes L_1 and L_2 determined for a simple geometrical figure – rhombus. It can be seen that the intersection of the axes and rhombus sides creates four rectangular triangles. The angle β can be expressed using the length of axes:

$$\tan\beta = \frac{\frac{1}{2}L_2}{\frac{1}{2}L_1} = \frac{L_2}{L_1} \tag{15}$$

Using (13) the criteria elongation M_3 can be obtained as following

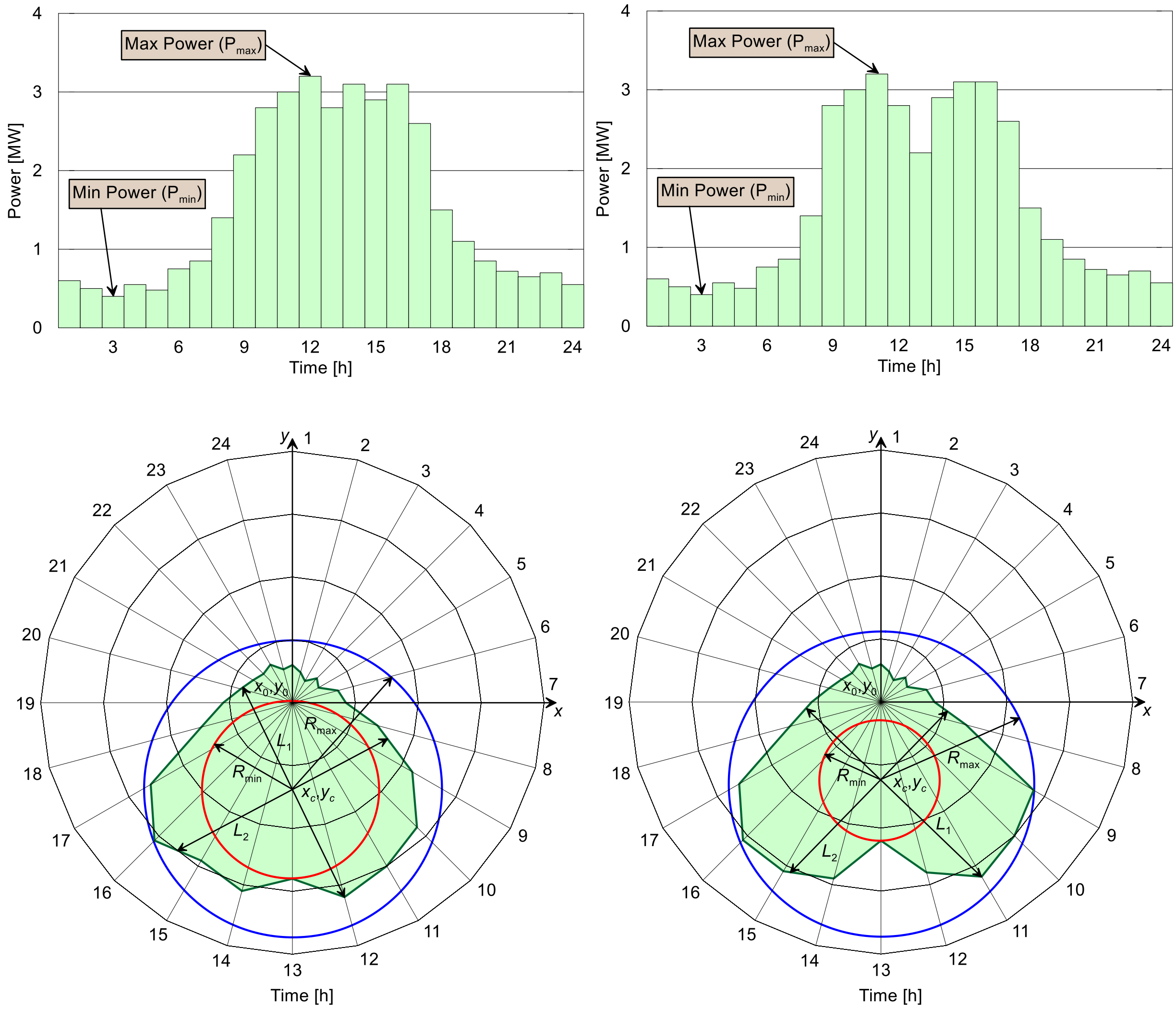
$$M_3 = \tan\beta \tag{16}$$

Fig. 4b shows the plot of M_3 against angle β. It can be seen that when the angle β and the length L_2 are growing the value of M_3 is increased. This confirms the tangential form of the function M_3 over the region $\beta \in (0; \pi/4)$. The simple cases of the elongation analysis (M_3) for regular polygons are given in Table 2.

Both Fig. 4b and Table 2 demonstrate that the value M_3 is approaching 1, if the angle β is growing to 45°. Fig. 7 shows that at β = 45° the general angle, which is divided by the main axis like bisector and equal to 2β, is equal to 90°. Therefore, this case is corresponding to $L_1 = L_2$ and $M_3 = 1$. The last row of Table 2 shows the derivative of the elongation. The derivative values are related to the speed of the function change $M_3' = \frac{1}{\cos^2\beta}$ in the example points.

6 Case Study

This section gives a description of the case study on application of the morphometric approach for the analysis of the daily load profiles. It underpins the explanation of the morphometric method and implementation of the relevant criteria for evaluation of the daily load profiles. The case study is based on analysis of two different daily load profiles (a) and (b) shown in Fig. 8.



(a) (b)

Fig. 8. Analysis of two daily load profiles with different patterns using morphometric method.

Initially both load profiles are represented as time series having m = 24 readings and analysed using the conventional indexes – average power P_{AVER}; load factor LF; non-uniformity index K_n and standard deviation σ. These indexes are calculated as following

$$P_{AVER} = \frac{1}{m}\sum_{i=1}^{m} P_i \tag{17}$$

$$LF = \frac{P_{AVER}}{P_{MAX}} \tag{18}$$

$$K_n = \frac{P_{\min}}{P_{\max}} \tag{19}$$

$$\sigma = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(P_i - P_{AVER}\right)^2} \tag{20}$$

The values of the indexes for the load profiles (a) and (b) are given in Table 3. It can be seen that numerical values of these parameters are exactly the same – it looks as if there is no difference between two load profiles; both time series have the same uniformity. This example demonstrates that the conventional indexes (17)-(20), in some cases, can not provide effective evaluation of the load profile pattern difference.

The conversion of the daily load profiles into the polygons in the radar charts makes it suitable for the morphometric analysis. In order to calculate the values of roundness M_1 and elongation M_3 the coordinates of the weight centre for each polygon are determined using (3). The weight centre of each polygon is the centre point for the inner radius $R_{\min}$ and outer radius $R_{\max}$, whereas the roundness M_1 is the ratio $R_{\min}/R_{\max}$ (4). The ideal value of the roundness $M_1 = 1$ is corresponding to a uniformed load profile. Table 3 shows the values of M_1 for both load profiles where the roundness for the profile (a) [$M_{1(a)} = 0.6154$] is greater than the roundness for the profile (b) [$M_{1(b)} = 0.3823$]. It means that the load profile (a) is a better uniformed profile than profile (b).

Table 3. Results of the load profiles' analysis for the case study.

	Load Profile (a)	Load Profile (b)
Average Power P_{AVER} [MW]	1.5542	1.5542
Load Factor LF	0.486	0.486
Non-uniformity index K_n	0.125	0.125
Deviation σ [MW]	1.0536	1.0536
Polygon Weight Centre (x_c; y_c)	−0.0835; −1.3839	−0.0048; −1.3144
Outer Radius R_{max}	2.3214	2.4285
Inner Radius R_{min}	1.4286	0.9285
Roundness M_1	0.6154	0.3823
Polygon Area S	10.7136	10.6049
Polygon Perimeter P	12.5576	13.2655
Compactness M_2	0.8536	0.7573
Length L_1	3.7142	3.8572
Length L_2	3.86	3.4285
Elongation M_3	1.0393	0.889

The area S and perimeter P for each polygon forms the compactness M_2 as shown in (8). In accordance to the definition of the criterion compactness its optimum value $M_2 = 1$ is related to the uniformed load profile. It can be seen from Table 3 that the compactness for the profile (a) [$M_{2(a)} = 0.8536$] is bigger and closer to $M_2 = 1$ than the compactness for the profile (b) [$M_{2(b)} = 0.7573$]. This demonstrates that the uniformity of the profile (a) is higher than (b).

The lengths of the lines L_1 and L_2 drawn through the weight centre are used to calculate the elongation M_3, which is the ratio L_2/L_1 defined in (13). In terms of profile variation, the elongation

M_3 is approaching 1 if the load profile is uniformed. Table 3 shows that the elongation for the profile (a) [$M_{3(a)}$ = 1.0393] is closer to 1 than the elongation for the profile (b) [$M_{3(b)}$ = 0.889]. This indicates that the uniformity of the load profile (a) is slightly better than the uniformity of the profile (b).

Therefore, the results of the morphometric analysis show that the load profile (a) has better uniform properties than the profile (b). This case study demonstrated that implementation of three morphometric criteria is a better choice for the evaluation of the daily load profiles' variation compared to the analysis using the conventional indexes.

7 Conclusion

The morphometric analysis of a power load profile is a new method based on graphical representation of the time series of the load readings as a polygon in a radar chart. The method focuses on the analysis of the geometric properties of the polygon in order to provide the load profile evaluation. This paper discusses the principles and implementation of three morphometric criteria (roundness M_1, compactness M_2, elongation M_3) applied for analysis of a daily load profile in the form of radar chart. The principles of these morphometric criteria have been explained using the examples where the load profiles were represented as a regular polygon. The morphometric analysis of the simple regular polygons has shown that the criterion roundness M_1 has a cosine character and its value is growing if the number of the polygon edges is increased. Despite the different definition, the criterion compactness M_2 shows exactly the same performance as the criterion roundness M_1. The third criterion elongation M_3 is related to angle β obtained from a rectangular triangle created by the intersection of the axes of the polygon. In the case of the regular polygon, the elongation value is growing with the increase of the angle β and approaching 1 at β = 45°.

In order to ensure a better understanding of the morphometric criteria implementation for the daily load profile analysis and to underpin the method explanation the case study has been given where two different load profiles were investigated. The case study demonstrated that the analysis using the discussed morphometric criteria provides better evaluation of the profile patterns in comparison to the conventional approaches.

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